STD : XI CHAPTER : 1 YEAR: 2012

MATHEMATICAL REASONING

STATEMENTS:-

A sentence is called a statement, if it is either true or false but not both an its truth or falseness can be definitely decided upon without ambiguity. It is also called a mathematically acceptable statement.

SIMPLE STATEMENT:-

A statement which cannot be broken into two statements is called a simple statement.

NEGATION:-

If a statement p is true, its negation would be false and vice versa, Negation of p is denoted by ~p.

If p is true ,we say it has truthvalue T and If p is false ,we say it has truthvalue F.

COMPOUND STATEMENT USING CONNECTIVES:-

Some simple statements are combined to yield a new statement called a compound statement.

Connectives : 'OR' and 'AND'

CONJUNCTION:- Conjunction of p and q is denoted by symbol $p \wedge q$.

DISJUNCTION:- Disjunction of p or q is denoted by symbol $p \vee q$.

INCLUSIVE SENSE:- i.e. In higher secondary science stream a student can opt for mathematics or biology. (A student can opt fir both the subject.)

EXCLUSIVE SENSE:- i.e. Bharat will go abroad for further studies or will study advanced mathematics in India immediately after passing 12^{th} standard. (Both the action cannot take place at the same time.)

QUANTIFIERS AD NEGATIONS:-

The use of phrases like 'there exists' and 'for all' or 'for every' is abundant in mathematics. These phrases are called quantifiers.

The phrases	symbol	Name of phrases
'there exists' and 'for all'	$\forall$	Universal quantifier
'for every'	$\exists$	Existential quantifier

Negations of Universal quantifier and Existential quantifier:-

Let p be any statement.

statement	symbol
~(for all p) = there exist ~p.	$\sim(\forall p) = \exists(\sim P)$
~(there exist p) = for all ~p.	$\sim(\exists p) = \forall(\sim P)$

Implication (Conditional) statement:- 'if p, then q' is called implication and is denoted by $p \Rightarrow q$.

When p is true and q is false then $p \Rightarrow q$ is false and true otherwise.

Double Implication (Biconditional) statement:- 'p if and only if q' is called double implication and is denoted by $p \Leftrightarrow q$.

When $p \Leftrightarrow q$ is true if both p and q are true or both are false and false otherwise.

Equivalent statement:- $p \Rightarrow q$ is equivalent to $\sim q \Rightarrow \sim p$.

Contrapositive statement:- $\sim q \Rightarrow \sim p$ is contrapositive statement of $p \Rightarrow$.

Converse of $p \Rightarrow q$ is $q \Rightarrow p$.

Rule : (1) $\sim(\sim p) = p$

(2) $\sim(p \wedge q) = (\sim p) \vee (\sim q)$

(3) $\sim(p \vee q) = (\sim p) \wedge (\sim q)$

(4) $p \Rightarrow q = (\sim p) \vee q = \sim q \Rightarrow \sim p$

(5) $\sim(p \Rightarrow q) = p \wedge (\sim q)$

(6) $(p \Leftrightarrow q) = (q \Leftrightarrow p)$

$= (p \Rightarrow q) \wedge (q \Rightarrow p)$

$= (\sim p \vee q) \wedge (\sim q \vee p)$

(7) $\sim(p \Leftrightarrow q) = (p \wedge \sim q) \wedge (q \wedge \sim p)$

$= p \Leftrightarrow (\sim q)$

$= q \Leftrightarrow (\sim p)$

N = the set of all natural numbers. = {1,2,3,4, … }

Z = the set of all integers. = {… -2, -1, 0,1 ,2 , …}

Q = the set of all rational numbers.

R = the set of real numbers.

(1) Listing Method (Roster Form):- i.e. N={1,2,3,…}

(2) Property Method (Set Builder Form):- We have denoted by

{x/P(x) }={x/The property of x}

If we write , M= {x/x is an integer, $-2 < x < 3$ } = { - 1 , 0 , 1, 2}

(3) A set consisting of only one element is called a singleton.

(4) A set which does not contain any element is called an empty set (or null set)

An empty set I denoted by { } or $\varnothing$.

(5) A set which is not empty is called a **non-empty set**.

(6) Generally when we consider many sets of similar nature, the element in the sets are selected from a definite set. This set is called the **universal set** and it is denoted by **U.**

(7) A set A is said to be subset of a set B if every element of A is also an element set B and is denoted by $A \subset B$.

Logical notation :- $(\forall x, x \in A \Rightarrow x \in B) \Rightarrow A \subset B$.

If set has n element the number of subsets is 2^n.

Theorem 2.1 $A \subset A$. Theorem 2.2 For any set A, $\varnothing \subset A$.

(8) Any non-empty set has at least two subset namely $\varnothing$ and the set itself. These subsets are called improper subsets. Other subsets (if any) of a set are called proper subset.

If set has n (n>1) element the number of proper subsets is $2^n - 2$.

(9) If set A is a subset of a set B, then set B is called a superset of A.

(10) For any set A, the set consisting of all the subset of a is called the power set of A and it is denoted by P(A) or 2^A.

$P(A) = \{ B / B \subset A \}$. If set has n element the number of power set is 2^n.

The power set of any set is never an empty set.

(11) Subsets of set of real numbers:- $N \subset Z \subset Q \subset R$

(12) The set of all irrational numbers:- $I = \{ x / x \in R, x \notin Q \}$

i.e. I is the set of real numbers that are not rationals.

(13) Interval:- $a, b \in R$

$(a, b) = \{ x / x \in R, a < x < b \}$ (open interval)

$[a, b] = \{ x / x \in R, a \le x \le b \}$ (closed interval)

$[a, b) = \{ x / x \in R, a \le x < b \}$ (closed-open interval)

$(a, b] = \{ x / x \in R, a < x \le b \}$ (open-closed interval)

$[0, \infty)$ = Set of non-negative real numbers.

$(- \infty , 0]$ = Set of negative real numbers.

$(- \infty, \infty) = R$

$(a, \infty) = \{ x / x \in R, x > a \}$

$[a, \infty) = \{ x / x \in R, x \ge a \}$

$(- \infty, a) = \{ x / x \in R, x < a \}$

$(- \infty, a] = \{ x / x \in R, x \le a \}$

(14) Equal sets:- If $A \subset B$ and $B \subset A$ then $A = B$.

OR If $\forall x , x \in A \Rightarrow x \in B$ and $\forall x , x \in B \Rightarrow x \in A$ then $A = B$.

(15) Operation of Sets:- (a) Union:- $A \cup B = \{ x / x \in A \text{ or } x \in B \}$

(b) Intersection:- $A \cap B = \{ x / x \in A \text{ and } x \in B \}$

(c) Complementation:- $A' = \{ x / x \in U \text{ and } x \notin A \}$

(d) Difference set:- $A - B = \{ x / x \in A \text{ and } x \notin B \}$

(e) Symmetric Difference set:- $A \Delta B = (A \cup B) - (A \cap B)$

(16) De Morgan's Laws:-(a) $(A \cup B)' = A' \cap B'$

(b) $(A \cap B)' = A' \cup B'$

	Union	Intersection

1. Binary Operation	$(A \cup B) \in P(U)$	$(A \cap B) \in P(U)$
2.	$A \subset (A \cup B)$ and $B \subset (A \cup B)$	$(A \cap B) \subset A$ and $(A \cap B) \subset B$
3. Idempotent law	$A \cup A = A$	$A \cap A = A$
4. If $A \subset B$ and $C \subset D$ then	$(A \cup C) \subset (B \cup D)$	$(A \cap C) \subset (B \cap D)$
5. Commutative law	$A \cup B = B \cup A$	$A \cap B = B \cap A$
6. Associative law	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
7. Identity element	$A \cup \varnothing = A$ thus $\varnothing$ is identity element	$A \cap U = U$ thus U is identity element
8.	$(A \cup U) = U$	$(A \cap \varnothing) = \varnothing$

(17) Distribution laws:-

Intersection operation over union operation	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Union operation over Intersection operation	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(18) Disjoint Set :- Non-empty sets A and B are said to be disjoint if their intersection is the empty set.

⊕ An Important Result:-If $A \subset B$ then $A \cup B = B$ and $A \cap B = A$.

	Complementation	Difference set	Symmetric Difference set
1	$A' \in P(U)$	$A - B = A \cap B'$	$A \Delta B = (A \cup B) - (A \cap B)$
2	$A \cap A' = \varnothing$, $A \cup A' = U$	$A - B \neq B - A$	$A \Delta B = (A \cap B') \cup (B \cap A')$
3	$\varnothing' = U$, $U' = \varnothing$	$U - A = A'$	$A \Delta B = (A \cup B) \cap (A \cap B)'$
4	$(A')' = A$	$A \subset B \Rightarrow A - B = \varnothing$	$A \Delta B = (A - B) \cup (B - A)$
			$A \Delta B = B \Delta A$

(19) Cartesian Product of Sets:-Let A and B be two non-empty sets. Then the set of all ordered pairs (x, y), where $x \in A$ and $y \in B$ is called Cartesian product of A and B. It is denoted by $A \times B$. (read: ′A cross B′)

⊕ $A \times B = \{ (x, y) / x \in A, y \in B\}$

⊕ If A or B or both are empty sets then we take $A \times B = \varnothing$.

⊕ $A \times A = A^2$.

⊕ $A \times B \times C = \{ (x, y, z) / x \in A, y \in B, z \in C\}$

(20) Number of elements of a Finite Set:-

Number of elements in a finite set A = n(A).

⊕ If A and B are disjoint set then $n(A \cup B) = n(A) + n(B)$

⊕ If $A \cap B = B \cap C = A \cap C = \varnothing$ then $n(A \cup B \cup C) = n(A) + n(B) + n(C)$

⊕ If $A \cap B \neq \varnothing$ then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

⊕ If $A \cap B \neq \varnothing$ then $n(A) = n(A - B) + n(A \cap B)$

⊕ If $A \cap B \neq \varnothing$ then $n(B) = n(B - A) + n(A \cap B)$

CHAPTER:- 3

RELATIONS AND FUNCTIONS

(1) Relation:- For any non-empty sets A and B, a subset of $A \times B$ is called a relation from A to B.

⊕ $n(A) = m$ and $n(B) = n \Rightarrow n(A \times B) = mn$

⊕ The number of subsets of $A \times B = 2^{mn}$, hence 2^{mn} relations are possible from A to B.

Neighbourhood : If $p, a, b \in R$, $a < b$ and $p \in (a, b)$ then (a, b) is called a neighbourhood of p.

δ-neighbourhood of a : - $N (a, \delta) = (a - \delta, a + \delta)$, $a \in R$, $a \in R$, $\delta > 0$. (mid point of segment)

Deleted δ-neighbourhood : - $N^*(a, \delta) = (a - \delta, a) \cup (a, a + \delta)$, $a \in R$, $\delta > 0$.

$$N^*(a, \delta) = \{ x / 0 < | x - a | < \delta, x \in R \}$$

Important Result : - (1) $| x | \le a \Leftrightarrow -a \le x \le a, x \in R, a \in R^+$.

(2) $| x | \ge a \Leftrightarrow x \le -a$ OR $x \ge a$.

(3) $(x - a)(x - b) \le 0 \Leftrightarrow a \le x \le b$.

(4) $(x - a)(x - b) > 0 \Leftrightarrow x \le a$ OR $x \ge b$.

(5) $| x - a | < \delta \Leftrightarrow a - \delta < x < a + \delta$.

$N(a, \delta) = (a - \delta, a + \delta)$ → Interval form

$= \{ x / a - \delta < x < a + \delta, x \in R \}$ → Inequality form

$= \{ x / 0 \le | x - a | < \delta \}$ → Modulus form

Some properties of δ - neighbourhood : -

(1) If $0 < \delta_1 < \delta_2$, then $N (a, \delta_1) \subset N (a, \delta_2), a \in R$.

(2) If $a \neq b$; $a, b \in R$, there exists a neighbourhood $N (a, \delta_1)$ of a and a neighbourhood $N (b, \delta_1)$ of b such that $N (a, \delta_1) \cap N (b, \delta_2) = \emptyset$.

(3) There is a δ-neighbourhood of p contained in any neighbourhood of p.

→ if (a, b) is a neighbourhood of p and $\delta = \min \{ p - a, b - a \}$

then $(p - \delta, P + \delta) \subset (a, b)$. Thus $N (p, \delta) \subset (a, b)$.

Fundamental Principle of counting : → (Principle)

If an event can occur in n different ways, followed by another event which can occur in m different ways then the total number of different ways in which both the events can occur in nm.

Linear Permutation : → A liner permutation of n objects taken r ($1 \le r \le n$) at a time is an arrangement in a line, in a definite order, of r objects taken at a time, from n distinct object, and is denoted by ${}_np_r$.

$${}_np_r = {}^np_r = P (n, r) = n(n - 1)(n - 2)(n - 3) \dots (n - r + 1) = \frac{n!}{(n-r)!}.$$

→ $nPn = n!$

→ The product of the first n natural numbers, that is $1 \cdot 2 \cdot 3 \cdot \dots \cdot (n - 1) \cdot n$

Is called n-factorial and is denoted n! OR ∟n.

→ $n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n, \quad 0! = 1$

Permutation with Repetition : → The number of ways of arranging n thing in m places, repetition being allowed, is n^m.

Permutations of Identical Objects : → Of the given n objects, p_1 are identical, p_2 are also identical but different from the p1 objects. Lastly p_k object are identical, but different from the pervious ones, and $p_1 + p_2 + p_3 + \ldots + P_k = n$. Then the number of distinct permutations of these n objects is $\frac{n!}{p_1! \cdot p_2! \cdot p_3! \cdot \ldots . p_k!}$.

Circular Permutation : → An ordered arrangement on a circle is called a circular permutation.

→ There are $(n-1)!$ Circular permutations of n objects, taken all at a time.

Combination : → An r-combination of n different objects is a selection of r objects at a time, out of n, irrective of the order of the selection.

→ $\binom{n}{r} = {}_{n}C_{r} = {}^{n}C_{r} = C(n, r) = \frac{n!}{(n-r)!r!}, 0 \leq r \leq n.$

→ $\binom{n}{n} = \binom{n}{0} = 1$

→ $\binom{n}{r} = \binom{n}{n-r}$

→ $\binom{n}{r} + \binom{n}{n-1} = \binom{n+1}{r}$

→ The value of $\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \ldots$ first increase and decrease. If n is even, then $\binom{n}{r}$ is maximum at $r = \frac{n}{2}$ and then the values decrease in the same order in which they occurred earlier.

If n is odd, then $\binom{n}{\frac{n-1}{2}}, \binom{n}{\frac{n+1}{2}}$ are equal and maximum. There after, the values decrease in the same order.

→ For given n, r ∈ N, $\binom{n}{r}$ = k, k ∈ N dose not have more then two solution.

For example, $\binom{4}{r} = 15$ has no solution, $\binom{4}{r} = 6$ has only one solution, r =2. $\binom{4}{r} = 4$ has two solution, r =1 and r = 2.

Pascal's Triangle : →

0	1						
1	1	1					
2	1	2	1				
3	1	3	3	1			
4	1	4	6	4	1		
5	1	5	10	10	5	1	
6	1	6	15	20	15	6	1

The numbers in Pascal triangle are precisely the same numbers as the coefficients of binomial expansion.

$(x+y)^0 = 1$ — 0th row

$(x+y)^1 = 1x + 1y$ — 1st row

$(x+y)^2 = 1x^2 + 2xy + 1y^2$ — 2nd row

$(x+y)^3 = 1x^3 + 3x^2y + 3xy^2 + 1y^2$ — 3rd row

$(x+y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$ — 4th row

$(x+y)^5 = 1x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + 1y^5$ — 5th row

Pascal's Triangle [Magic 11's]

If a row is made into a single number by using each element as a digit of the number (carrying over when an element itself has more than one digit), the number is equal to 11 to the nth power or 11^n when n is the number of the row the multi-digit number was taken from.

Row #	Formula	=	Multi-Digit number	Actual Row
Row 0	11^0	=	1	1
Row 1	11^1	=	11	1 1
Row 2	11^2	=	121	1 2 1
Row 3	11^3	=	1331	1 3 3 1
Row 4	11^4	=	14641	1 4 6 4 1
Row 5	11^5	=	161051	1 5 10 10 5 1
Row 6	11^6	=	1771561	1 6 15 20 15 6 1
Row 7	11^7	=	19487171	1 7 21 35 35 21 7 1
Row 8	11^8	=	214358881	1 8 28 56 70 56 28 8 1

Binomial Theorem : → n ∈ N,

→ $(a+b)^n = \binom{n}{a} a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + \binom{n}{n} b^n$

→ $(a-b)^n = \binom{n}{a} a^n - \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + (-1)^r \binom{n}{r} a^{n-r}b^r + \ldots + (-1)^n \binom{n}{n} b^n$

→ $(1+x)^n = \binom{n}{0} + \binom{n}{1} x + \binom{n}{2} x^2 + \ldots + \binom{n}{r} x^r + \ldots + \binom{n}{n} x^n$

→ $T_{r+1}(\text{Term}) = \binom{n}{r} a^{n-r}b^r,\ 0 \le r \le n$

→ If n is even, the $\left(\frac{n}{2}+1\right)^{th}$ term is middle one.

→ If n is odd, then there are two middle terms, $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$.

→ If a constant term, the index of x is zero.

→ The Binomial coefficients: $\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \ldots \binom{n}{n}$.

→ $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots + \binom{n}{n} = 2^n, \forall\, n \in N.$

→ $\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \ldots = \binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \ldots = 2^{n-1}, \forall\, n \in N$

COMPLEX NUMBERS : -

The Set R X R is the set of ordered pairs of real number $R \times R = \{ (a, b) / a \in R, b \in R \}$

1. Equality : $a = c, b = d \Rightarrow (a, b) = (c, d)$

2. Addition : $(a, b) + (c, d) = (a + c, b + d)$

3. Multiplication : $(a, b).(c, d) = (ac - bd, ad + bc)$

The set R X R with these rules is called the set of complex numbers and it is denoted by C.

Generally, we denote a complex number by z.

Properties :-

	Algebraic	Multiplication
1.Closure	$z_1 + z_2 \in C$	$z_1.z_2 \in C$
2.Commutative	$z_1 + z_2 = z_2 + z_1$	$z_1.z_2 = z_2.z_1$
3. Associative	$(z_1 + z_2) + z_3 =$ $z_1 + (z_2 + z_3$	$(z_1 z_2) z_3 =$ $z_1 (z_2 z_3)$
4. Identity	(0,0) is called additive identity or zero	(1, 0) is called multiplicative

	complex number. $z + 0 = z = 0 + z$ The additive identity 0 is unique.	identity. $z(1, 0) = z = (1, 0)z$ The multiplicative identity (1, 0) is unique.
5. Inverse	The complex number z = (a , b) the corresponds complex number (- a, - b), denoted by – z called the additive inverse of z. $z + (-z) = 0$	The non-zero complex number z = (a , b) the corresponds complex number $\left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$ denoted by z^{-1} called the multiplicative inverse of z. $z \cdot z^{-1} = (1,0) = z^{-1}.z$ z^{-1} is denoted by $\frac{1}{z}$.
6.The distributive laws	(a) $z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3$ (b) $(z_1 + z_2)z_3 = z_1 z_3 + z_2 z_3$	

→ $N \subset Z \subset Q \subset R \subset C$

→ R is a Subset of C.

→ In the year 1737 Euler was first person to introduce the symbol i for the complex number.

→ i = (0 , 1) is called imaginary number.

→ Representation of Complex number :-

$$z = (x, y) = x + iy = Re(z) + iIm(z) = r(\cos\theta + i\sin\theta)$$

- (x , y) is order pair. $x \in R$, $y \in R$
- x is called real part of complex number and is denoted by Re(z).
- y is called imaginary part of complex number and is denoted by Im(z).
- $x = r\cos\theta$ and $y = r\sin\theta$. Here $r = \sqrt{x^2 + y^2}$, $-\pi < \theta \le \pi$.

Two Complex Number : $z_1 = a + bi$, $z_2 = c + di$

Equality of two Complex Number	$z_1=z_2 \Rightarrow (a,b) = (c,d) \Rightarrow a=c$ and $b=d$
Addition of two Complex Number	$z_1+z_2=(a,b) + (c,d) =(a+c,b+d)$
Difference of two Complex Number	$z_1-z_2=(a,b) - (c,d) =(a-c,b-d)$
Multiplication of two Complex Number	$z_1.z_2=(ac-bd, ad+bc)= (ac-bd)+(ad+bc)i$
Quotient of two Complex number	$\frac{z_1}{z_2} =z_1.z_2^{-1}$
Powers of i	<table><tr><td></td><td>In general ↓</td></tr><tr><td>$i=\sqrt{-1}$</td><td>$i^{4k} = 1$</td></tr><tr><td>$i^2 = -1$</td><td>$i^{4k+1} = i$</td></tr><tr><td>$i^3= -i$</td><td>$i^{4k+2} = -1$</td></tr><tr><td>$i^4 = 1$</td><td>$i^{4k+3} = -i$</td></tr></table>
Conjugate of a Complex Number	$\bar{z} =a-bi=(a,-b)$
Modulus of a Complex Number	$\|z\|=\sqrt{a^2+b^2}$

Properties of Conjugate Complex Number :

1. $\overline{(\bar{z})}= z$
2. $\frac{z+\bar{z}}{2} = \text{Re}(z)$
3. $\frac{z-\bar{z}}{2} = \text{Im}(z)$
4. $z = \bar{z}$ *if and only if z is real.*
5. $\bar{z} = -z$ if and only if z is purely imaginary.
6. $\overline{z_1 \pm z_2} = \overline{z_1} \pm \overline{z_2}$
7. $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$
8. $\overline{\left(\frac{z1}{z2}\right)} = \frac{\overline{z1}}{\overline{z2}}$, where $z2 \neq 0$

Properties of Modulus:

1. $|z| = 0$ if and only if $z = 0$.
2. $|z| \geq |\mathrm{Re}(z)|$; $|z| \geq |\mathrm{Im}(z)|$
3. $z\bar{z} = |z|^2$
4. $|z| = |\bar{z}|$
5. $|z| = |-z|$
6. $\dfrac{z_1}{z_2} = \dfrac{z_1\overline{z_2}}{|\overline{z_2}|^2}$, where $z_2 \neq 0$
7. $|z_1 z_2| = |z_1||z_2|$
8. $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$, where $z_2 \neq 0$
9. $|z_1 + z_2| \leq |z_1| + |z_2|$
10. $|z_1 - z_2| \geq \big||z_1| - |z_2|\big|$

→ $z = r(\cos\theta + i\sin\theta)$ is called poler form complex number z. Also θ is known as amplitude or argument of z. It is denoted by arg(z).

→ Value of θ satisfying $x = r\cos\theta$ and $y = r\sin\theta$; $-\pi < \theta \leq \pi$ is knownas the principal valueof arg(z)

→ Argument of complex number 0 is not defined.

→ $\arg(x + i\,0) = \begin{cases} 0, & \text{if } x > 0 \\ \pi, & \text{if } x < 0 \end{cases}$

→ $\arg(0 + i\,y) = \begin{cases} \dfrac{\pi}{2}, & \text{if } y > 0 \\ -\dfrac{\pi}{2}, & \text{if } y < 0 \end{cases}$

→ Argument of positive real number is 0 and that of negative real number is π.

→ Argument of purely imaginary number yi is $\dfrac{\pi}{2}$ and $-\dfrac{\pi}{2}$ according as $y > 0$ or $y < 0$ resp..

Square Root of a Complex number : $\rightarrow$

The square roots of $z = x + iy$ are $\pm\left(\sqrt{\frac{|z| + x}{2}} - i\sqrt{\frac{|z| + x}{2}}\right)$

$\rightarrow$ The square root of 1 are $\pm$ 1.

$\rightarrow$ The square root of -1 are $\pm$ i.

Qudratic Equation having a Complex Roots : $\rightarrow$

$ax^2 + bx + c = 0$ is Quadratic Equation $\Rightarrow D = b^2 - 4ac$

$\rightarrow$ If $D > 0$, root of $ax^2 + bx + c = 0$ are $\frac{-b \pm \sqrt{D}}{2a}$.

$\rightarrow$ If $D = 0$, root of $ax^2 + bx + c = 0$ are $\frac{-b}{2a}$.

$\rightarrow$ If $D < 0$, root of $ax^2 + bx + c = 0$ are $\frac{-b \pm i\sqrt{D}}{2a}$.

Fundamental Theorem of Algebra :$\rightarrow$

Every polynomial equation having complex coefficient and degree ≥ 1 has at least one coplex root.

Cube Roots of Unity :–

The cube roots of unity are 1, $\frac{-1+\sqrt{3}i}{2}$, $\frac{-1-\sqrt{3}i}{2}$.

Properties of Cube Roots of Unity :

1. Each of two non–real cube roots of unity is the square of each other.

   Let $\omega = \frac{-1+\sqrt{3}i}{2}$. Then $\omega^2 = \frac{-1-\sqrt{3}i}{2}$.

   Hence cube roots of unity are 1, ω, ω^2.

2. The sum of cube roots unity is 0. i.e. $1 + \omega + \omega^2 = 0$.

3. The product of cube roots unity is 1. i.e. $1 \times \omega \times \omega^2 = 1$.

4. Representating 1, $\frac{-1+\sqrt{3}i}{2}$, $\frac{-1-\sqrt{3}i}{2}$ in the Argand plane as A,B,C respectively, Thus A,B,C are the vertices of an equilateral triangle.

$\rightarrow$ In general, nth root of 1 forms a regular polygon of n sides, with vertices on the unit circle.

$\rightarrow$ Multiplication of z b I produces rotation of an angle of measure $\frac{\pi}{2}$.

QUADRATIC EQUATIONS : -

$ax^2 + bx + c = 0$, $a \neq o$, a, b, c $\in$ R is called a quadratic equation in variable x.

→ Δ is called discriminat of quadratic equation, $\Delta = b^2 - 4ac$.

→ If $\Delta > 0$ then two real roots α, β. Where $\alpha, \beta = \frac{-b \pm \sqrt{\Delta}}{2a}$.

→ If $\Delta = 0$ then $\alpha = \beta = \frac{-b}{2a}$.

→ If $\Delta < 0$ then two complex roots α, β. Where $\alpha, \beta = \frac{-b \pm i\sqrt{\Delta}}{2a}$.

Nature of the Roots using Discriminant :

(1) If $\Delta > 0$ it has positive square root. The roots α, β are real and unequal.

(2) If a, b, c are rational numbers and Δ is a square of a non-zero rational number, then α and β are rational and unequal.

(3) If $\Delta = 0$ then $\alpha = \beta = \frac{-b}{2a}$.

(4) If $\Delta < 0$ it does not have a real square root, but it does have two non-real complex square roots. In this case α and β are complex conjugate numbers.

Sum and Product of Roots : -

The roots of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, a, b, c $\in$ R are α and β

$$\therefore \alpha + \beta = \frac{-b}{a}, \qquad \alpha\beta = \frac{c}{a}$$

→ The quadratic equation whose roots are α and β is $x^2 +(\alpha+\beta)x+\alpha\beta=0$

Symmetric Expressions of Roots : -

→ $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

→ $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

→ $|\alpha - \beta|^2 = \left|\sqrt{(\alpha - \beta)^2}\right| = |(\alpha - \beta)^2|$

→ $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

→ $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$

Type of roots on the basis of coefficients:-

1. $b = 0$, the roots are additive inverse, Hence $\alpha + \beta = 0$.

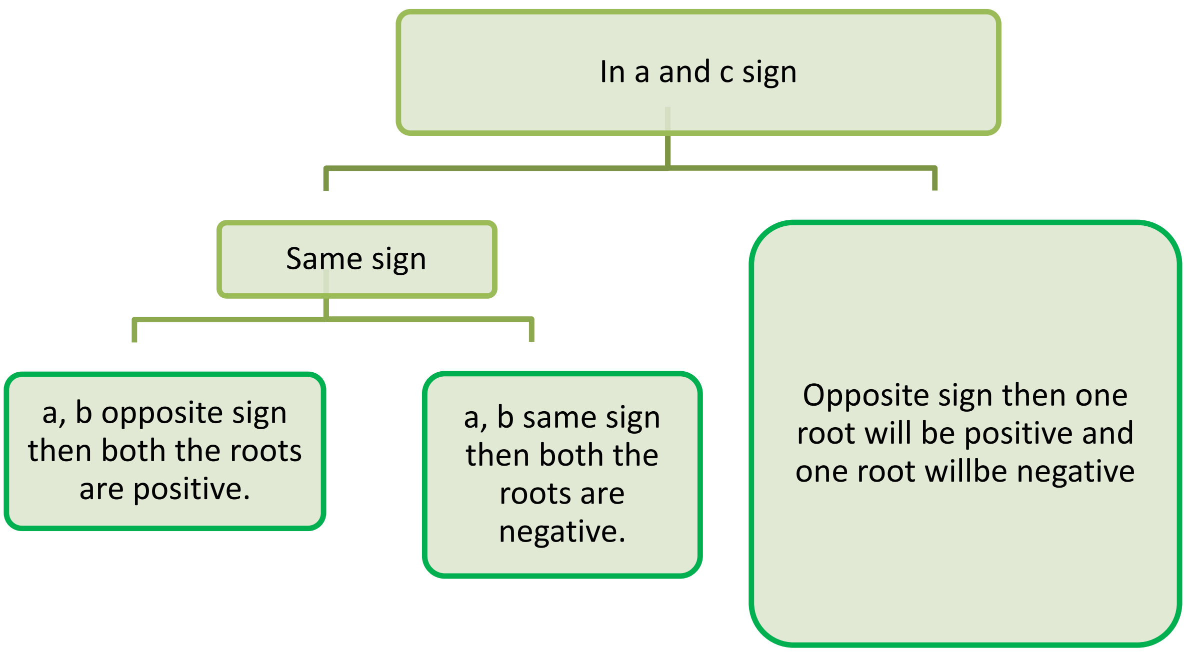
2. $a = c$, the roots are multiplicative inverse, Hence $\alpha\beta = 1$.

3. a, b and c have the same sign, Both the roots are negative.

4. $c = 0$ then one root is zero and second root is $\frac{-b}{a}$.

5. $b = c = 0$ then both roots are zero.

6.



EXPONENTIAL FUNCTION: -

Let $a \in R^+$. Then the function $f : R \rightarrow R^+$, $f(x) = a^x$ is called the exponential function. a is called the base of the function.

Properties of Exponential Function: - let $a > 0$, $a \neq 1$, $f(x) = a^x$, $x \in R$

(1) The domain of f is R, its codomain and range are R^+.

(2) f is one-one. i. e. $a^x = a^y \Rightarrow x = y$.

(3) f is onto R^+.

(4) f is increasing function for $a > 1$.

(5) f is decreasing function for $0 < a < 1$.

We shall accept the following properties of real exponents.

Let $a, b \in R^+$: $x, y \in R$. we have (1) $a^x \cdot a^y = a^{x+y}$

(2) $\frac{a^x}{a^y} = a^{x-y}$

(3) $(a^x)^y = a^{xy}$

(4) $(ab)^x = a^x b^y$

(5) $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$.

A Special exponential function: -

$f : R \rightarrow R^+$, $f(x) = e^x$, where e is an irrational number (like π) is a special exponential function which plays an important role in higher mathematics.

Approximate value of e is 2.71828… .

Logarithmic Function: -

Let $a \in R^+ - \{ 1 \}$ and f be the exponential function with the base a,

$f = \{ (x, y) / y = a^x, a \in R^+ - \{ 1 \}, x \in R, y \in R^+ \}$. Then the inverse of f,

$f^{-1} = \{ (y, x) / y = a^x, a \in R^+ - \{ 1 \}, x \in R, y \in R^+ \}$ is called the logarithmic function to the base a and it is denoted by $\log_a$.

Properties of the Logarithmic Function: -

(1) The domain of logarithmic function is R+ and its range is R.

(2) The exponential function and the logarithmic function are inverse of each other.

$$y = f (x) \Leftrightarrow x = f^{-1}(y), \quad \therefore y = a^x \Leftrightarrow x = \log_a y.$$

(3) For any $a \in R+ - \{ 1 \}$, $a^0 = 1 \Leftrightarrow \log_a 1 = 0$.

(4) The logarithmic function is one-one on R^+.

(5) The logarithmic function is onto on R.

(6) $\log_a x$ is an increasing function for $a > 1$.

(7) $\log_a x$ is an decreasing function for $0 < a < 1$.

(8) The logarithm to the base 10 is called common logarithm.

(9) The logarithm to the base e is natural (Napierian) logarithm.

Properties of Logarithm: - $a, b \in R^{+} - \{ 1 \}$: $x, y \in R^{+}$; $x_1, x_2, x_3, \dots, x_n \in R^{+}$;

(1) $a^{\log_a x} = x \ (x > 0)$

(2) $\log_a a^x = x \ (x \in R)$

(3) Product Rule:- $\log_a(xy) = \log_a x + \log_a y$

Corollary: - $\log_a (x_1 + x_2 + x_3 + \dots + x_n) = \log_a x_1 + \log_a x_2 + \log_a x_3 + \dots + \log_a x_n$.